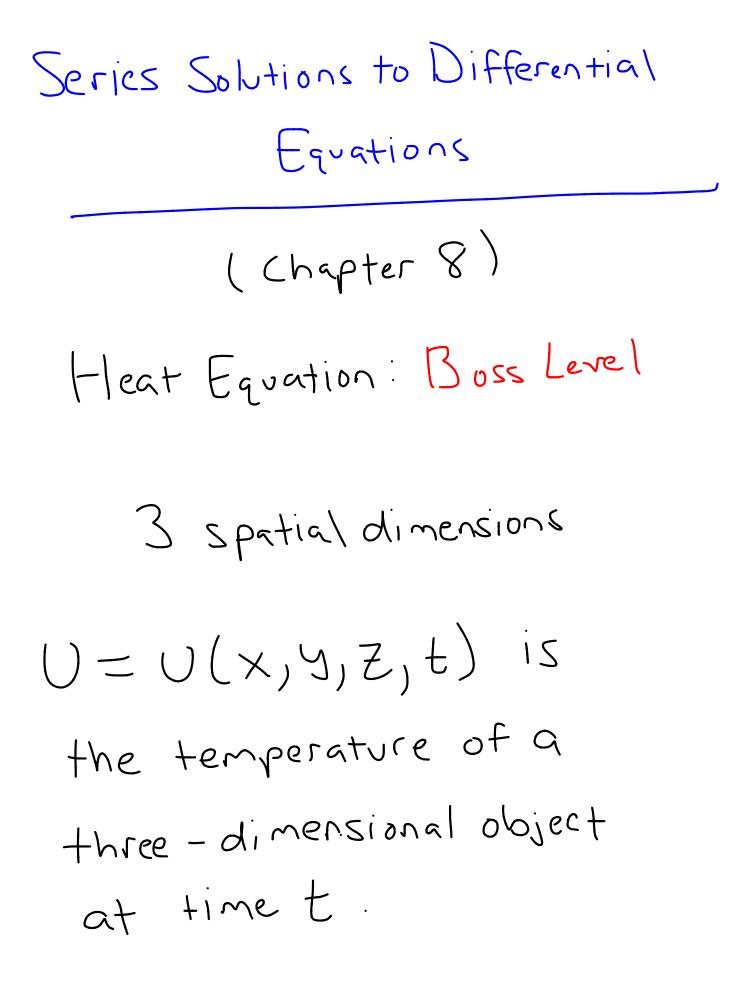
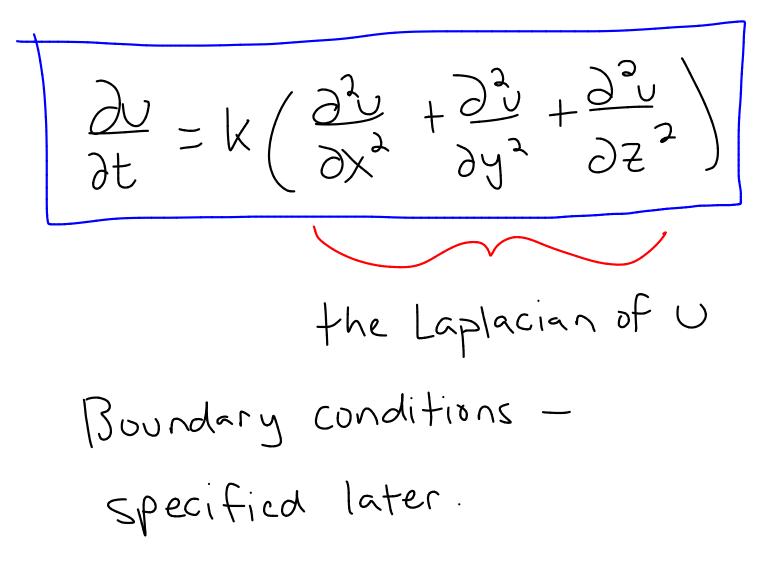
Announcements

## 1) Quiz tomorrow over section 7.8



Heat Equation



lemperature in a Cylinder

$$X^{+}y^{-} \leq 10$$

Cylindrical to Cartesian  $X = \Gamma COS \Theta \qquad \qquad \Theta = \arctan\left(\frac{y}{x}\right)$   $Y = \Gamma SIN\Theta \qquad \qquad \Gamma = \sqrt{x^2 + y^2}$   $Z = Z \qquad \qquad \qquad Z = Z$ 

Convert Iteat Equation to cylindrical coordinates

We get  $\frac{\partial v}{\partial t} = k \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial^2 v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right)$ 

Assume temperature is independent of 7 and 0. Then  $\partial U = \partial U = O$ , so we get  $\frac{\partial V}{\partial v} = K \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial v} \right)$ 

We suppose  $U(\Gamma, \Theta, Z, t)$ = u(r,t)= f(r)g(t)Now apply the heat equation:  $\frac{\partial U}{\partial r} = f'(r)g(t), \frac{\partial u}{\partial r^2} = f'(r)g(t)$ 

 $\frac{\partial U}{\partial t} = f(r)g'(t)$ 

We get q'(t)f(r) $= k \left( f''(r)g(t) + \frac{1}{r}f'(r)g(t) \right)$  $=kg(t)(f'(r) + \frac{1}{r}f(r))$ Divide both sides by kf(r)g(t)  $\frac{1}{k} \frac{g'(t)}{g(t)} = \frac{f'(r) + \frac{1}{r} f(r)}{f(r)}$ 

 $\frac{1}{k} \frac{g'(t)}{g(t)} = \frac{f'(r) + \frac{1}{r} f(r)}{f(r)}$ 

Since we have a function of r equal to a function of to both sides of the equality must be constant:

 $\frac{1}{k} \frac{g'(t)}{g(t)} = \frac{f''(r) + \frac{1}{r} f(r)}{f(r)}$   $= -\alpha \qquad for some$ real number  $\alpha$ 

We get two ordinary differential equations  $\frac{1}{4} \frac{g'(t)}{g(t)} = -\alpha' easy$ 

 $f''(\iota) + \frac{\iota}{\iota} f(\iota) = - \prec$ f(r)

Rewriting,  $f''(r) + \frac{r}{l} f'(r) = -f(r)$ 

h)e have 

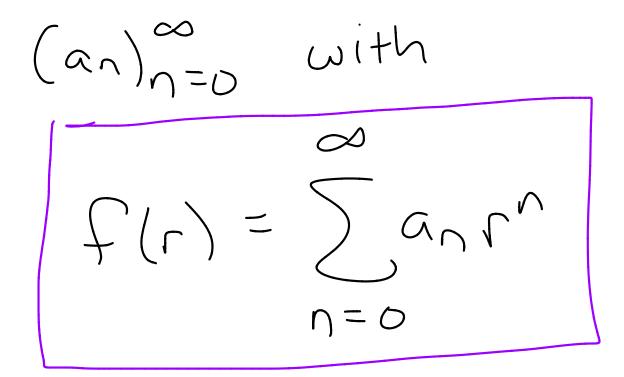
homogeneous, second-order, Nonconstant coefficients

By theory, there is a solution! I tow to find it?

Not Cauchy-Euler!

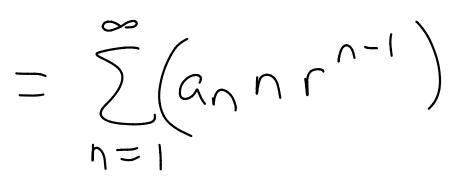
Wishful Thinking

Assume there are real numbers



purely formal!

$$f'(r) = \sum_{n=0}^{\infty} a_n(nr^{n-1})$$



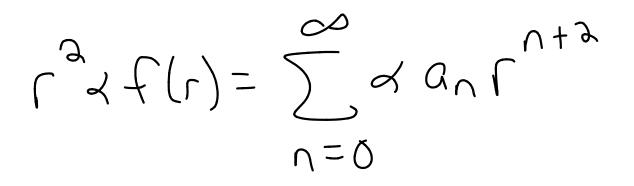
$$f''(n) = \sum_{n=1}^{\infty} a_n \left( \left( n - 1 \right) n^{-2} \right)$$

$$= \sum_{n=2}^{\infty} (a_n (n-1) (n-2))$$

to make the indices nice, multiply  $f''(r) + \frac{1}{r}f'(r) + \alpha f(r) = 0$ by r2 (not r]) to get  $l_{r}(t) + l_{r}(t) + d_{r}(t) = 0$ 

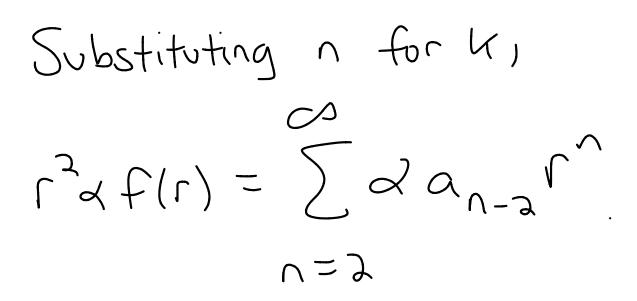
$$\Gamma^{2}\Gamma''(r) = \sum_{n=2}^{\infty} (a_{n}n(n-1)r^{n})$$

$$\Gamma^{2}\Gamma'(r) = \sum_{n=2}^{\infty} (a_{n}nr^{n})$$



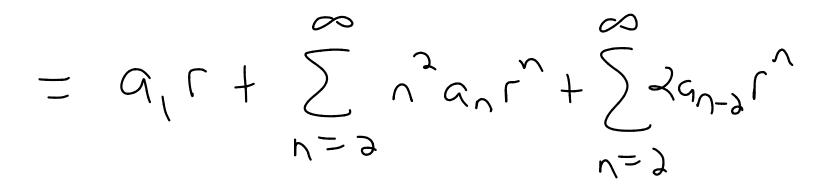
reindex with k=n+2, then n=k-2 $r^{2} \downarrow f(r) = \sum_{k=2}^{\infty} \downarrow a_{k-2} r^{k}$ k=2

 $r^2 \chi f(r) = \sum \chi q_{k-2} r^k$ イーン



lhe  $L_{a}t_{n}(t) + tt_{n}(t) + d_{b}t(t)$  $= \sum_{n=2}^{\infty} (a_n(n)(n-1)r^n)$  (ombine  $+ \sum_{n=1}^{\infty} (a_n n r^n)$ ろい  $\sum_{n=2}^{n=2}$  leave alone

We get  $\alpha_{1}\Gamma + \sum_{n=1}^{\infty} \left( n(n-1) + n \right) \alpha_{n}\Gamma^{2}$  $+ \sum_{n=2}^{\infty} \alpha_{n-2} r^{n}$ 



 $= q_{1}r + \sum_{n=1}^{\infty} (n q_{n} - d q_{n-2})r^{n}$